Proof and Meaning

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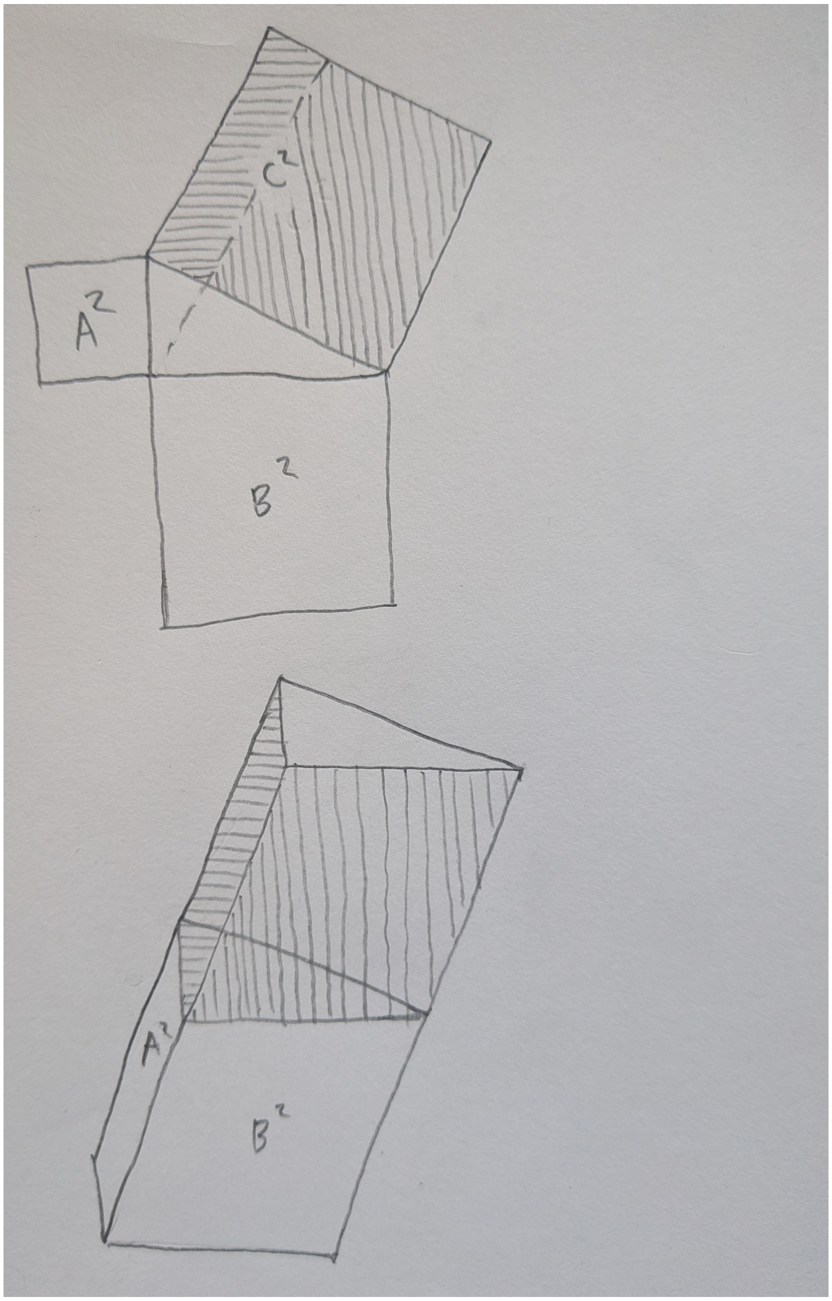
1 Introduction

There is a difference between proving a theorem and knowing why it is true. The hardest proof I ever read was the proof from "Baby" Rudin that if an ordered set has the least upper bound property, then it also has the greatest lower bound property. I was able to confirm each statement followed logically from the proceeding statements. But because the abstract idea of an ordered set was used instead of easier spaces like subsets of the real number field, I had no idea why the result was true, even though I know it is.

2 Examples of Theorems without Intuition

If a proof does not give us intuition, this encourages us to search for better proofs. The following are examples of well known theorems, that would be nicer to understand.

The first example is the Pythagorean theorem that has many different proofs. The common example of rearranging 4 triangles inside of a square area is not as intuitive as the following proof using the idea the shearing does not change the area of a quadrilateral.



Other examples include:

The pull on an electron between charged parallel plates is the same everywhere if the plates are long relative to the distance of the electron to the edge. This is easy to proof using integration, but harder to understand why, which prompts us to seek a more clear proof.

If we add unimodal sinusoids of the same frequency we always get another unimodal sinusoid. The are several ways to prove this using geometry or complex numbers. But non of them that I have seen give any intuition. It would be interesting to do something like what Shilov does with sinusoids in his book of functional analysis, and to ask instead, what kinds of waves have the property that if we add unimodal waves, we get a unimodal wave?