π is constant

Alex Glandon

July 2026

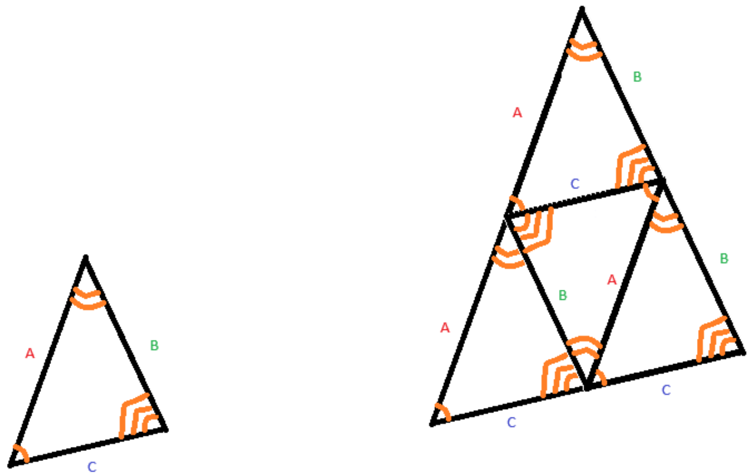
1 Introduction

If we want to show that all circles have the same ratio of their circumference to their diameter, we can proceed as follows. We can approximate a circle as a regular polygon. For example, if we approximate a circle with diameter D as a square with edges of length D, this is not a very good approximation, and the ratio of the perimeter of the circle to the diameter is 4. If we increase the number of sides of the polygon, the ratio would become closer to π. We could write a formula for the series as a function of the number of sides n, and check that the limit converges, and that it gives us a value of π. This would require that the ratios of the edges of the polygons to the diameter not be affected by the size of the polygons. If for example, a regular octagon is built using 8 isosceles triangles with two edges of length D and one edge of length E, where E is the length of the edge of the octagon, then we would need a lemma to show that the ratio of E to D is constant for any size triangle to complete our proof of that π is a constant for any size circle.

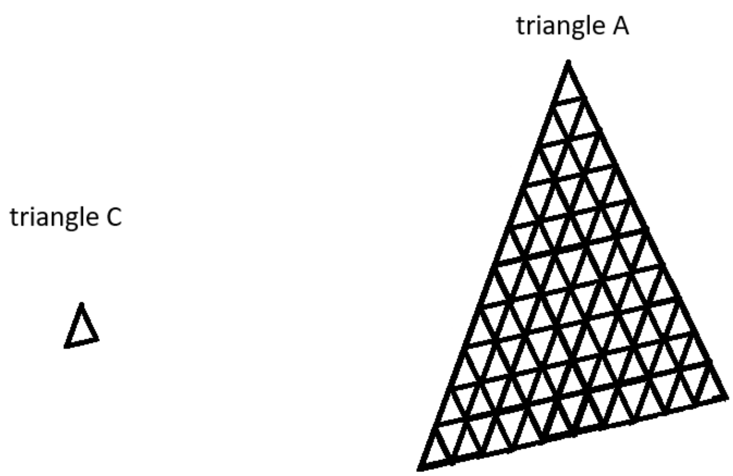
2 Triangle Lemma

For example, if a triangle has the same angles (as for example when we divide the 360 degrees of a circle into 8 pieces and we obtained triangles of 360/8=45 degrees toward the center of the octagon and 67.5 degrees for both angles toward the edges of the octagon), we would like to show that the ratio of any two sides is the same no matter what the size of the triangle.

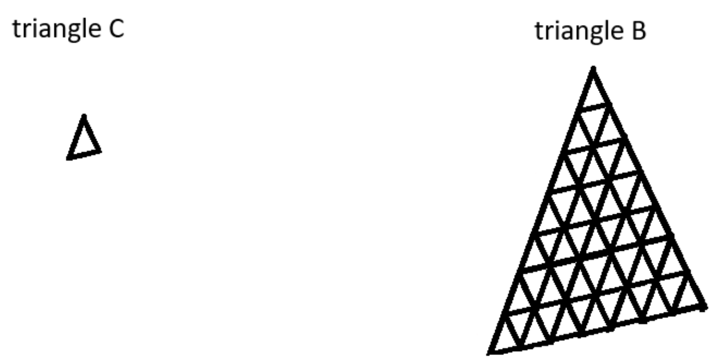
First lets us show that if the triangle is twice the size, that the ratios are the same as in the figure below. We can see that all sides in the larger triangle are twice the size of the smaller triangle given that the angles are kept the same.



Next let use show for any ratio $\frac{n}{m}$ between the sizes of two triangles, if the angles are the same, that the ratios of the sides are the same.



nC = A



mC = B

Since $nC = A$, $mC = B$, then $C = \frac{B}{m}$, and $A = \frac{n}{m}B$.