Guessing Game Solution

Alex Glandon

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1 Introduction

The following paradox (see next page) was posted on Fermat's library. We propose a solution as follows:

Assuming a number from $\mathbb{R}$ can be selected without any human bias.

Player one has picked two numbers A and B.

A is the visible number (the card seen by player 2).

B is the invisible number (the card not seen by player 2).

The number player 2 picked before seeing anything was named T.

If $A < T$, player 2 guesses $A < B$.

If $A > T$, player 2 guesses $A > B$.

We can include for completeness a third case, if $A = T$, then player 2 can have a second backup splitting number U, where again if $A < U$ guess $A < B$ and if $A > U$ guess $A > B$. This can be applied using our proposed solution without loss of generality.

Chapter in *Open Problems in Communication and Computation.*
Springer-Verlag, 1987. T. Cover and B. Gopinath, editors., P. 152

5.1. PICK THE LARGEST NUMBER

Thomas M. Cover

Departments of Electrical Engineering
and Statistics
Stanford University
Stanford, CA 94305

Player 1 writes down any two distinct numbers on separate slips of paper. Player 2 randomly chooses one of these slips of paper and looks at the number. Player 2 must decide whether the number in his hand is the larger of the two numbers. He can be right with probability one-half. It seems absurd that he can do better.

We argue that Player 2 has a strategy by which he can correctly state whether or not the other number is larger or smaller than the number in his hand with probability *strictly greater than one-half.*

Solution: The idea is to pick a random *splitting number* T according to a density $f(t)$, $f(t) > 0$, for $t \in (-\infty, \infty)$. If the number in hand is less than T, decide that it is the smaller; if greater than T, decide that it is the larger.

Problem: Does this result generalize? Does it apply to the secretary problem?

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(solution on next page)

Now given the setup, we can calculate the probability of player 2 winning.

$P(\text{player 2 win}) =$

$P([\text{player 2 guesses } A < B \text{ AND } A < B]$ OR
$[\text{player 2 guesses } A > B \text{ AND } A > B]) =$

$P(\text{player 2 guesses } A < B \text{ AND } A < B) +$
$P(\text{player 2 guesses } A > B \text{ AND } A > B)$
(note that the probability of either events occurring given that they are mutually exclusive is the sum of the individual probabilities, and we will use fact this below)

Notice that:

$P(\text{player 2 guesses } A < B \text{ AND } A < B) =$
$P(A < T \text{ AND } T < B) + P(A < B \text{ AND } B < T)$
Because player 2 only guesses $A < B$ if $A < T$.

Again:

$P(\text{player 2 guesses } A > B \text{ AND } A > B) =$
$P(A > T \text{ AND } T > B) + P(A > B \text{ AND } B > T)$
Because player 2 only guesses $A > B$ if $A > T$.

For both cases above, we assume $P(A = T) = 0$, and do not include that possibility, as the article also omits given we assumed no bias in human selection of numbers from $\mathbb{R}$.

Finally, there are two cases:

case 1: $A < B$

$P(\text{player 2 win}) =$

$P(\text{player 2 guesses } A < B \text{ AND } A < B) +$
$P(\text{player 2 guesses } A > B \text{ AND } A > B) =$

$P(A < T < B) + P(A < B < T) + P(A > T > B) + P(A > B > T) =$

$P(A < T < B) + P(A < B < T) + 0 + 0 =$

$P(A < T) =$

0.5

case 2: $A > B$

$P(\text{player 2 win}) =$

$P(\text{player 2 guesses } A < B \text{ AND } A < B) +$
$P(\text{player 2 guesses } A > B \text{ AND } A > B) =$

$P(A < T < B) + P(A < B < T) + P(A > T > B) + P(A > B > T) =$
$0 + 0 + P(A > T > B) + P(A > B > T) =$
$P(A > T) =$
0.5

So in either case, the probability of player 2 winning is 50%.