

Generalization of the Pigeonhole principle

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1 Arithmetic Generalization

$$\sum_{i=1}^N a_i = A \implies \exists a_i \geq \frac{A}{N}$$

$$\sum_{i=1}^N a_i \geq A \implies (\text{same}) \exists a_i \geq \frac{A}{N}$$

$$\sum_{i=1}^N a_i > A \implies \exists a_i > \frac{A}{N}$$

2 Geometric Generalization

all b_i positive

$$\prod_{i=1}^N b_i = B \implies \exists b_i \geq \sqrt[N]{B}$$

$$\prod_{i=1}^N b_i \geq B \implies (\text{same}) \exists b_i \geq \sqrt[N]{B}$$

$$\prod_{i=1}^N b_i > B \implies \exists b_i > \sqrt[N]{B}$$