

Fuel Loop Problem

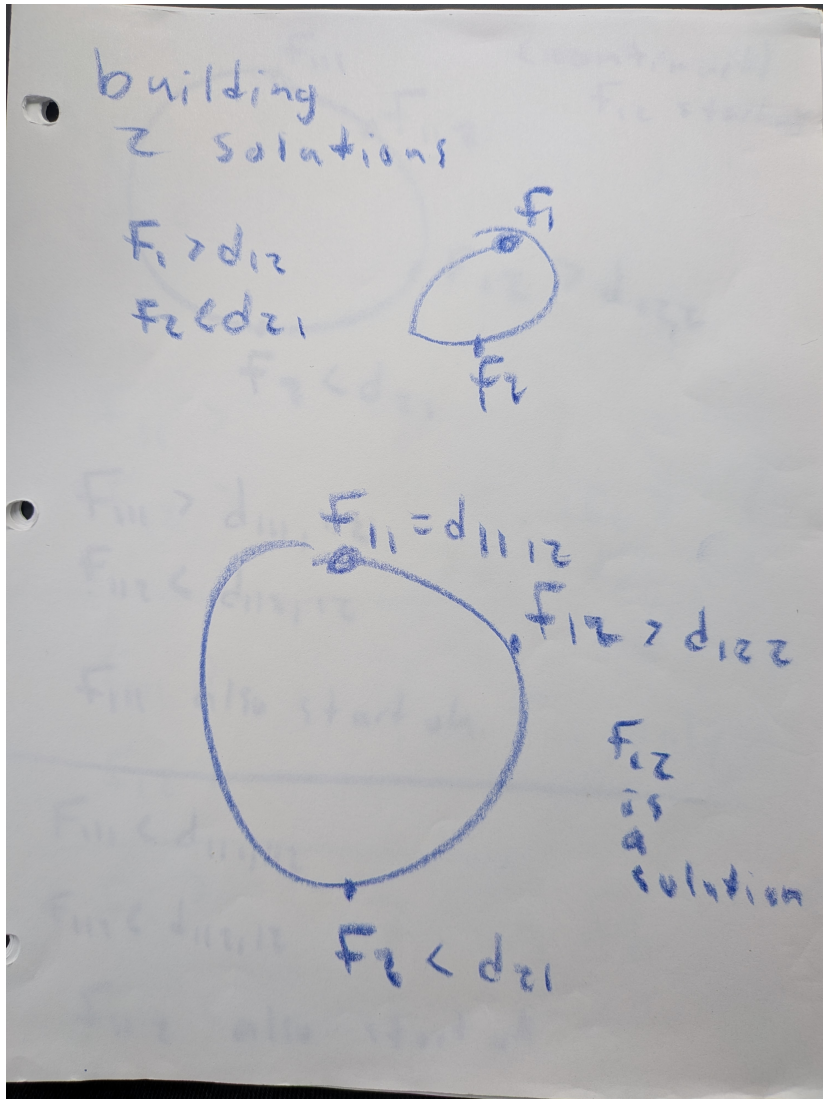
Alex Glandon

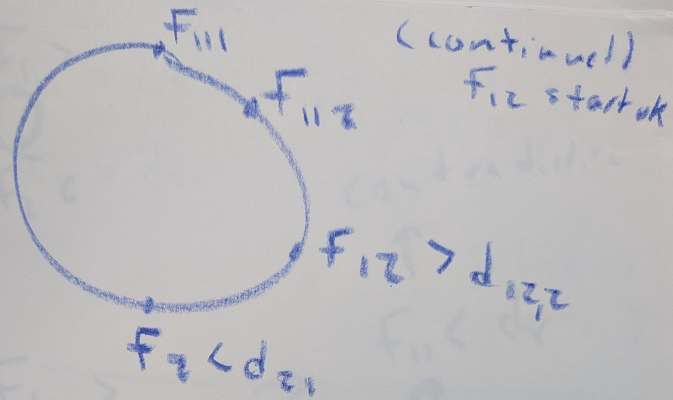
May 2026

1 Problem

A set of fuel tanks is distributed in a cyclic path. The sum of the fuel from all tanks equals the total fuel needed. Prove that the path is traversable starting with an empty vehicle tank.

2 Notes / Proof Sketch





$F_{111} > d_{111,112}$
 $F_{112} < d_{112,112}$

F_{111} also start uk

$F_{112} < d_{111,112}$

$F_{112} < d_{112,112}$

F_{112} also start uk

$$F_{11} > Q$$

$$F_{12} < Q$$

Can take direction
 ↑ solution
 $F_{11} < dx$

$$F_{11} > ?$$

$$F_{12} > ?$$

$$F_{12} + F_{22} > ?$$

$$dx + dx$$

$$F_{11} > ?$$

$$F_{12} = ?$$

→ F_{11} only

$$F_{11} > ?$$

$$F_{12} < ?$$

→ F_{11} only

$f_1 = d_{12}$

$f_2 = d_{21}$

$f_{11} = d_{12}$
 $f_{12} = d_{12}$

(3
solutions
starts ↗

$f_1 = d_{12}$

$f_2 = d_{21}$

$f_{11} > d_{12}$

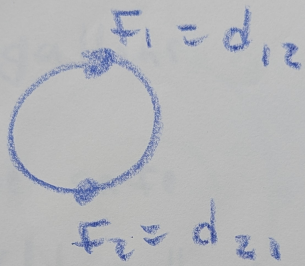
$f_{12} < d_{12}$

f_{11}, f_2
start
solutions

f_1 only
starting
point

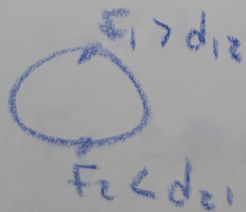
reverse induction
(build the problem)

type A



F_1, F_2 starting points

type B



F_1 only
starting
point

applied problem

every mile is

G gallons

to be safe

$G \times 1.1$ gallons

total fuel is

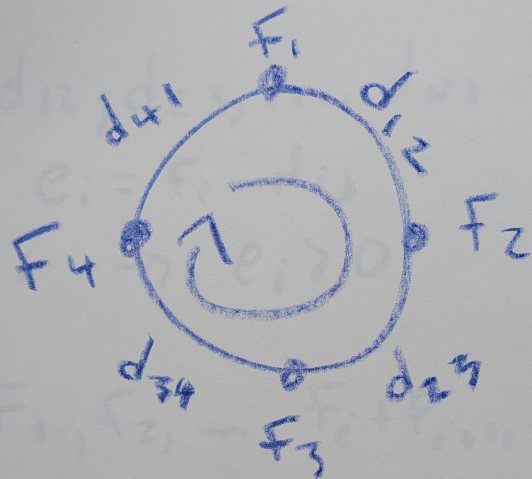
$F \times 1.1$

safety \rightarrow

$$F_i \geq 1.1 \times d_{ij}$$

\rightarrow same solution

DOCUMENT EXPLICIT SOLUTION



$$\sum F_i = F$$

$$\sum d_{ij} = F$$

reduction (clockwise)

$$F_1, F_2, \dots, F_N$$

$$d_{12}, d_{23}, \dots, d_{N1}$$

$$e_i = f_i - d_i$$

$$\exists i \rightarrow e_i > 0$$

$$F_1, F_2, \dots, F_i + F_{i+1}, \dots, F_N$$

$$d_{12}, d_{23}, \dots, d_{ij} + d_{i+1, j+1}, \dots, d_{N1}$$

induction

new problem

$$\exists i \rightarrow e_i > 0$$