

Exponential Linear Region

Alex Glandon

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1 Introduction

Consider a system where use of the system is successful M percent of the time.

For example, suppose a system is success 75% of the time. The following table shows the chance of success for all trial of the system, given that the system is used N times.

N	system success
1	$(0.75)^1 = 75\%$
2	$(0.75)^2 = 56\%$
3	$(0.75)^3 = 42\%$

For example, suppose a system is success 90% of the time. The following table shows the chance of success for all trial of the system, given that the system is used N times.

N	system success
1	$(0.90)^1 = 90\%$
2	$(0.90)^2 = 81\%$
3	$(0.90)^2 = 73\%$
4	$(0.90)^2 = 66\%$
5	$(0.90)^2 = 59\%$
6	$(0.90)^2 = 53\%$
7	$(0.90)^2 = 48\%$

2 Linear Region Theorem

There is a strange phenomenon that occurs when the failure chance of a trial of the system F , defined as $1 - P(\text{success})$ is within a certain region.

For example, consider $F = 10^{-9}$. Then the chance of any system failure given

N trials can be simplified from $P(\text{failure}) = 1 - P(\text{success}) = 1 - (1 - F)^N$.

In case one, if $N < 10^9$, then $P(\text{failure}) \approx N \times 10^{-9}$

In case two, if $N > 10^9$, then $P(\text{failure}) \approx 1$.

For a second example, consider $F = 10^{-1}$.

In case one, if $N < 10^1$, then $P(\text{failure}) \approx N \times 10^{-1}$

In case two, if $N > 10^1$, then $P(\text{failure}) \approx 1$.

This leads to the general theorem:

Given failure chance of a single trial of a system $F = 10^{-x}$,

The chance of any failure in N trials given $N < 10^x$ is approximately $N \times 10^{-x}$.

The chance of any failure in N trials given $N > 10^x$ is approximately 1.

A more general theorem can be easily derived if we consider a system with Q components of failure chance F being tested R times by substituting $Q \cdot R$ for N above.