

Arithmetic Hacks

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2 digit number divided by 2

$$([a][b])/2$$

$$a \text{ even} \Rightarrow [\frac{a}{2}][\frac{b}{2}]$$

$$b \text{ even} \Rightarrow [\frac{a-1}{2}][\frac{10+b}{2}]$$

examples:

$$26/2 = [2/2][6/2] = 13$$

$$34/2 = [\frac{3-1}{2}][\frac{10+4}{2}] = [\frac{2}{2}][\frac{14}{2}] = 17$$

$[a][9] \times 3$

$$[a][9] \times 3 = [(a+1) \times 3][0] - b$$

examples:

$$19 \times 3 = [(1+1) \times 3][0] - 3 = 60 - 3 = 57$$

$$29 \times 3 = [(2+1) \times 3][0] - 3 = 90 - 3 = 87$$

modular division of many digit numbers

modular division can be done blockwise,

example:

$$178247\%7 = [17\%7][8\%7][247\%7]$$

(note block sizes are arbitrary)

$$[17\%7][8\%7][247\%7] = [3][1][002]$$

$$[31\%7][002]$$

$$[3][002]$$

$$[30\%7][02]$$

$$[2][02]$$

$$202\%7 = 6$$

therefore $178247\%7 = 6$

reference: Euler

recursive sum of digits to 3

If the digits of a number sum to 3, 6, or 9, then the number is divisible by 3. Using inverse recursion, it follows that if the digits of a number sum to a number whose digits sum to 3, 6, or 9, that number is also divisible by 3 and so on.

example: 5484, is divisible by three since $5 + 4 + 8 + 4 = 21$ and $2 + 1 = 3$.

reference: through the grapevine from my friend Shannon

(works in base 10 or larger radix r where $r-1$ is divisible by 3)

repeating patterns

$[xyz][xyz]...[xyz]$ is not prime

for example, 711711 is not prime, since we know there is at least a factor of 711, since 711711 is 711×1001

$9 \times n$

$$9 \times n = [n][0] - n$$

for example $15 \times n = 150 - 15 = 135$