Arithmetic Hacks

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2 digit number divided by 2

$([a][b])/2$

a even => $[\frac{a}{2}][\frac{b}{2}]$

b even => $[\frac{a-1}{2}][\frac{10+b}{2}]$

examples:

26/2=[2/2][6/2]=13

$34/2 = [\frac{3-1}{2}][\frac{10+4}{2}] = [\frac{2}{2}][\frac{14}{2}] = 17$

$[\mathbf{a}][\mathbf{9}] \times \mathbf{3}$

$[a][9] \times 3 = [(a+1) \times 3][0] - b$

examples:

$19 \times 3 = [(1+1) \times 3][0] - 3 = 60 - 3 = 57$

$29 \times 3 = [(2+1) \times 3][0] - 3 = 90 - 3 = 87$

modular division of many digit numbers

modular division can be done blockwise,

example:

178247%7 = [17%7][8%7][247%7]

(note block sizes are arbitrary)

[17%7][8%7][247%7] = [3][1][002]

[31%7][002]

[3][002]

[30%7][02]

[2][02]

202%7 = 6

therefore 178247%7 = 6

reference: Euler

recursive sum of digits to 3

If the digits of a number sum to 3, 6, or 9, then the number is divisible by 3. Using inverse recursion, it follows that if the digits of a number sum to a number whose digits sum to 3, 6, or 9, that number is also divisible by 3 and so on.

example: 5484, is divisible by three since $5 + 4 + 8 + 4 = 21$ and $2 + 1 = 3$.

reference: through the grapevine from my friend Shannon

(works in base 10 or larger radix r where r-1 is divisible by 3)

repeating patterns

$[xyz][xyz]...[xyz]$ is not prime

for example, 711711 is not prime, since we know there is at least a factor of 711, since 711711 is 711×1001

$9 \times$ n

$9 \times n = [n][0] - n$

for example $15 \times n = 150 - 15 = 135$

$[\mathbf{a}][\mathbf{b}] - \mathbf{c}$

if $b < c$: $[a][b] - c = [a-1][10 - (c-b)]$

example:

$23 - 7 = [1][10 - (7-3)] = [1][10-4] = [16]$

corollary:

$(a-b) + ([1][b] - a) = 10$

example

(7-3)+(13-7)=10

and more obviously:

(13-7)+(7-3)=10

curiosities

$b < 5$:

$[a][b] - 5 = [a-1][c]$

$\Leftrightarrow$

$[a][b] + 5 = [a][c]$

$b >= 5$:

$[a][b] - 5 = [a][c]$

$\Leftrightarrow$

$[a][b] + 5 = [a+1][c]$