# Basis Representation Theorem

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## 1 Statement

$\forall A \in \mathbb{N}$

Given $B \geq 2$, called the base,

There exists unique $C_1, C_2, ..., C_N$ all greater than or equal to 0 and all less than $B$ that solve

$A = C_1 \cdot B^N + C_2 \cdot B^{N-1} + ... + C_{N-2} \cdot B + C_{N-1}$

## 2 Proof

Find the largest $B^N$ such that $\frac{A}{B^N} = C_1 + \frac{D_N}{B^N}$ and $C_1 \geq 0$.

(if $C_1 \geq B$ then we know we haven't found the largest $B_N$)

Euclid's Lemma tells us that $C_1$ and $D_N$ are unique.

This can be rewritten as $A = C_1 \cdot B^N + D_N$.

Euclid's lemma also tells us that $0 \leq D_N < B^N$.

Next find the solution to $\frac{D_N}{B^{N-1}} = C_2 + \frac{D_{N-1}}{B^{N-1}}$ which Euclid's lemma tells us is unique if $0 \leq C_2$ and $0 \leq D_{N-1} < B^{N-1}$.

(because $D_N < B^N$ we know $C_2 < B$) from the above equation)

Repeat for $B^{N-2}, B^{N-3}, ..., B^0$ which gives unique $C_3, C_4, C_5, ...C_{N-1}$ all greater than or equal to 0 and all less than $B$.