

Introduction to Prime Numbers

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1 Theorem 1

$A \neq B$, A and B both prime

$$\{1, 2, 3, \dots, B\} = \{A \% B, (2A) \% B, (3A) \% B, \dots, ((B - 1)A) \% B\}$$

(as sets)

2 Theorem 2

$B < A$, A and B both prime

$$\forall C, \{1, 2, 3, \dots, B\} =$$

$$\{(A + C) \% B, (2A + C) \% B, (3A + C) \% B, \dots, ((B - 1)A + C) \% B\}$$

(for Ms. Shannon)

3 Lemma 1

There is only one even prime number, 2.

Proof by contradiction: Suppose there is a prime number that is larger than 2 and is also even. Because it is even, it is divisible by 2, and therefore it is not prime.

4 Theorem 3

If a number n is even, then any factorization of n into all prime factors will have 2 as one of the factors.

Proof by contradiction:

Suppose $n = l_1 \times l_2 \times l_3 \times \dots \times l_k$ where each factor is prime

Consider if each factor l_i was an odd number, i.e. not divisible by 2

Then $l_1 \times l_2$ is odd since any product of odd numbers is also odd

Then also $(l_1 \times l_2) \times l_3$ is also odd since $(l_1 \times l_2)$ is odd and l_3 is odd

If we continue this argument we see that $l_1 \times l_2 \times l_3 \times \dots \times l_k$ is odd and therefore $n \neq l_1 \times l_2 \times l_3 \times \dots \times l_k$

If the prime factors cannot be all odd, then at least one of the prime factors must be even. By lemma 1 we know the only even prime factor is 2, and therefore any factorization of an even number into primes has a factor of 2.