Definitions of Group

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1 Miller's Definition

A set of elements S and an operator $*$

Closure: $a \in S, b \in S \ \rightarrow a * b \in S$

Associativity: $(a * b) * c = a * (b * c)$

Identity: $\exists I$, such that $a * I = I * a = a$

Inverse: $\forall a, \exists b$ (called a^{-1}), such that $a * b = b * a = I$

2 Arden's Definition

A set of elements S and a set of unary operators T.

Closure: $\forall a \in S, E \in T, F \in T \rightarrow E(F(a)) \in S$ and $E \circ F \in T$

Associativity:

theorem: function composition is always associative given the definition of a function

Identity: $\exists I \in T, \forall a \in S, a = I(a)$

Inverse: $\forall E \in T, \exists F \in T, \forall a \in S, E(F(a)) = a$

3 Conjecture

Using set elements as the group elements and a binary operator is homeomorphic to using set elements and unary operators as the group elements.

(for GS Venkatesh)

(also for the author of PVS)